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The sound of a gun fired at B reaches M_1 at the time T, and M_2 at the time $T + \tau_1$ sec., and it reaches M_3 at the time $T + \tau_2$ sec. How far is B from M_1?

2874. Proposed by J. L. RILEY, Stephenville, Texas.

Show that the equation,

$$x^n + ax^{n-1} + bx^{n-2} + \cdots + k = 0,$$

has some imaginary roots if $a^2 - 2b < n\sqrt[n]{k^2}$; $a, b, \cdots k$ are supposed real.

Note. This result is a particular case of a theorem contained in a paper published a few years ago.—EDITORS.

2875.

Show how to draw through a given point a straight line dividing a given triangle into two parts of equal area.

Three cases of this well-known problem (proposed by an Association member) were discussed by Euclid over two thousand years ago. A new solution is sought. —EDITORS.

PROBLEMS—NOTES

1. Mr. R. M. Winger, of the University of Washington, suggested the problem: "Six of the points of the Feuerbach circle of a triangle, namely, the middle points of the sides of the triangle and the middle points of the junctions of the opposite vertices with the orthocenter, lie in pairs at the extremities of diameters." This result is the basis of several proofs that Feuerbach's circle goes through the six points noted above as well as through the feet of the altitudes. See, for example, J. Casey, *Sequel to . . . the Elements of Euclid*, third edition, 1884, p. 58.

H. P. M.

2. The following problem was proposed and solved in the *Journal of the Indian Mathematical Club*, 1910: "All the significant figures except four in a sum of division were replaced by dots, and the result was:

```
···)······(···
    ·0··
    ----
     ····
     ·50·
     ----
       ···
       ·4·
       ---
```

It is required to recover the lost figures."

Similar problems with "four fours" and "seven sevens" are due to Mr. W. E. H. Berwick, lecturer in University College, Bangor, England:

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···)······4(·4··          ····7·)··7·······(··7··
    ···                          ······
    ---                          ------
    ··4·                         ·····7·
    ····                         ·······
    ----                         -------
     ····                          ·7····
      ·4·                          ·7····
     ----                          ------
      ····                         ·······
      ····                         ····7··
      ----                         -------
                                     ······
                                     ······
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The first of these was proposed in *The Mathematical Gazette,* March, 1920, page 43, with the statement: "There are just four ways of filling up the missing figures so as to leave a correctly-worked long division sum (in scale ten)." The problem was reproduced in *The Observatory,* July, 1920, page 274, and the solutions were given on page 372 of the issue for October. The "seven sevens" problem was proposed in *The School World,* London, July 1906, volume 8, page 280; its solution appeared in the following August number, page 320. The proposer remarked that "there is one, and only one solution possible." ARC.

3. A member of the Association has proposed as a problem the derivation of the familiar formula:

$$\cos nx = 2^{n-1}\cos^n x - 2^{n-3}\,n\cos^{n-2}x + 2^{n-5}\frac{n(n-3)}{2\,!}\cos^{n-4}x - \cdots$$

$$+ (-1)^r\frac{2^{n-2r-1}n(n-r-1)(n-r-2)\cdots(n-2r+1)}{r\,!}\cos^{n-2r}x + \cdots$$

where n is a positive integer. Such a derivation may be found in many well known works, for example: R. Levett and C. Davison, *Elements of Plane Trigonometry,* London, 1892, pp. 299–300; S. L. Loney, *Plane Trigonometry,* Cambridge, 1893, pp. 323–354; J. A. Serret, *Traité de Trigonométrie,* 6e édition, Paris, 1880, pp. 204–236; T. J. I'A. Bromwich, *An Introduction to the Theory of Infinite Series,* London, 1908, p. 177; and H. Weber and J. Wellstein, *Encyklopädie der Elementar-Mathematik,* Band 2, 2te Auflage, Leipzig, 1907, pp. 320–321. In obtaining the result, DeMoivre's theorem, the expansion of log $(1-2p\cos x+p^2)$,[1] and mathematical induction are variously employed (cf. problem 2828, *1920,* 186).

References may be given also to discussions in Lagrange, *Lecons sur le Calcul des Fonctions,* seconde édition, Paris, 1806; in *Cambridge Mathematical Journal,* vol. 4, November, 1844, pp. 250–252, by R. Moon; and in *Nouvelles Annales de Mathématiques,* 2e série, vol. 12, 1873, pp. 408, 425, by Mourgue and LeBesgue; vol. 14, 1875, p. 385, by Desboves.

In a letter to Leibnitz dated June 16, 1676, Newton gave a formula which, in modern notation, may be written:

$$\sin nx = n\sin x + \frac{(1-n^2)n}{3\,!}\sin^3 x + \frac{(1-n^2)(9-n^2)n}{5\,!}\sin^5 x + \cdots.$$

About twenty-five years later Johann Bernoulli derived the formula which, in modern notation, may be written:

$$\cos nx = \cos^n x - \binom{n}{2}\sin^2 x\cos^{n-2}x + \binom{n}{4}\sin^4 x\cos^{n-4}x - \cdots.$$

The formula with which this note opens seems to have been first given in 1748 by Euler, who arrived at the result by mathematical induction. (See *Introductio in analysin infinitorum.* Tomus primus. Lausannae, 1748; French edition,

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[1] This method was used by E. Heine, *Mathematische Annalen,* vol. 2, 1870, pp. 187–188.

Paris, 1796, pp. 194–195. Also "Dilucidationes super formulis, quibus sinus et cosinus angulorum multiplorum exprimi solent, ubi simul ingentes difficultates diluuntur," *Nova acta acad. sc. Petrop.*, volume 9 (1791), 1795, pp. 54–55; "Conventuı exhib. die 6 Mart. 1777.") In a posthumous paper, "Enodatio insignis cujusdam paradoxi circa multiplicationem angulorum observati," (*Opera postuma*, vol. 1, 1862, pp. 299–314) Euler discussed cos $n\phi$ as a power series in cos ϕ.

The developments of sin nx and cos nx into series in sin x and cos x, when n is not restricted to integral values, seem to have been first thoroughly discussed by Poinsot and forms the subject of his *Recherches sur l'Analyse des Sections Angulaires*, Paris, 1825, 80 quarto pages. ARC.

SOLUTIONS OF PROBLEMS.

417 (Algebra) [1914, 156]. Proposed by A. J. RICHARDSON, Marquette, Mich.

Required to reduce the quartic

$$x^4 + px^2 + qx + r = 0$$

to the form

$$(x^2 + k)^2 = [(2k - p)x + (2k - q)]^2,$$

wherein k is the solution of a certain cubic.[1] Hence, express the solution of the given quartic in terms of p, q, r, and k.

SOLUTION BY H. S. UHLER, Yale University.

We shall show that the hypothesis that the given quartic can be reduced to the required form destroys the mutual independence of the coefficients p, q, r, and that k is not "the solution of a certain cubic."

Transposing the right-hand member of

$$(x^2 + k)^2 = [(2k - p)x + (2k - q)]^2,$$

expanding, and collecting terms, we find

$$x^4 + [2k - (2k - p)^2]x^2 - 2(2k - p)(2k - q)x + [k^2 - (2k - q)^2] = 0.$$

Since, by hypothesis, the last equation is a modified form of the given quartic the coefficients of like powers of x in the two equations must be equal, that is, the three following conditions must hold

$$2k - (2k - p)^2 = p, \tag{1}$$

$$- 2(2k - p)(2k - q) = q, \tag{2}$$

$$k^2 - (2k - q)^2 = r. \tag{3}$$

Condition (1) gives

$$2k - p = 0, \tag{4}$$

or

$$2k - p = 1. \tag{5}$$

In either case k is a linear function of p alone and hence k is not dependent upon the solution of any cubic.

Combining relations (2) and (4) we see that q must have the special value zero. Then the given quartic reduces to a quadratic in x^2 and the solution is immediate.

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[1] There must have been some mistake in writing this problem. The following may have been intended:

$$(x^2 + k)^2 = [x\sqrt{2k - p} - \sqrt{k^2 - r}]^2.$$

See Todhunter, *An Elementary Treatise on the Theory of Equations*, London, 1880, p. 117.—EDITORS.